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E-5331

M.A. / M.Sc. (IIIrd Semester) Examination, 2021

MATHEMATICS

(Integration Theory and Functional Analysis)

Time Allowed : Three Hours

Maximum Marks : 70

Note : Attempt the questions as per given instructions in each section.

SECTION - A

Note : Attempt all questions. **2×5=10**

Q. 1. (i) A measure μ is called _____ if it has a decomposition.

(ii) Each σ-bounded Baire set is the _____ of a countable disjoint union of bounded Baire sets.

(iii) There is a _____ Baire measure on X that agree with μ on all compact sets.

(iv) The Hahn decomposition is _____ except for null sets.

(v) A complete normed linear space is called _____.

SECTION - B

Note : Attempt any five questions. **2×5=10**

Q. 2. (i) Write statement of Jordan decomposition theorem for signed measure.

(ii) Prove that a finite disjoint union of inner regular sets of finite measure is inner regular.

(iii) Prove that the space ℓ_p is a normed linear space.

(iv) Write short note on mutually singular measure.

(v) Write short note on equivalent norms.

(vi) Define caratheodory outer measure.

(vii) Show that in a normed linear space, every convergent sequence is a Cauchy sequence.

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SECTION - C

Note : Attempt any five questions. **4×5=20**

Q. 3. (i) Let $x = (x_1, x_2, \ldots\ldots x_n)$ and $y = (y_1, y_2, \ldots\ldots y_n)$ be n-tuples of scalars and define the norm by :

$$\|x\| = \left[\sum_{i=1}^{n} |x_i|^2\right]^{1/2} \cdot \text{ Then } \sum_{i=1}^{n} |x_i\, y_i| \le \|x\| \cdot \|y\|$$

(ii) If μ_0 is a Baire measure and if for every C in ϕ $\lambda(C) = \inf\{\mu_0(V_0) : C \subset V_0\}$ then prove that λ is a regular content.

(iii) Let E be a measurable set of finite measure in (X, A, μ). Then prove that E contains a positive A with $\mu(A) > 0$.

(iv) If μ and v are two signed measure, then define $\mu << v$.

(v) Show that the space C [0, 1] of real valued continuous functions on [0, 1] is a normed linear space.

(vi) If N is a normed linear space and K a convex subset of N, then prove that K is also convex.

(vii) Show that a normed linear space is a metric space with $d(x, y) = ||x - y||$ and $d(x, 0) = ||x||$.

SECTION - D

Note : Attempt any three questions. **3×10=30**

Q. 4. (i) State and prove Riesz Lemma.

(ii) Suppose that (X, A_1, μ) and (Y, A_2, V) are σ-finite. Let E, F belongs to $A_1 \times A_2$ and $V(E_x) = V(F_x)$ for all $x \in X$. Then prove that $\mu(E) = \mu(F)$.

(iii) State and prove Riesz markoff theorem.

(iv) State and prove Radon Nikodym theorem.

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