I-295

M.A./M.Sc. (First Semester)

Examination, Dec. - Jan., 2024-25

MATHEMATICS

Paper - II

(Real Analysis - I)

Time Allowed : Three Hours

Maximum Marks : 70

Note : Attempt all sections as directed.

(SECTION - A)

(Objective/Multiple Choice Questions)

Note : Attempt all questions. Choose the correct answer. **1×10=10**

Q. 1. For each $n \in \mathbb{N}$, the sequence $\{x^n\}$ is :

(a) Uniformly convergent on $x \in [0, k]$, $k < 1$

(b) Uniformly convergent on $x \in [0, k]$, $k = 1$

(c) Uniformly convergent $\forall\, x \in \mathbb{R}$

(d) None of the above

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Q. 2. The series $\frac{\cos x}{1^p} + \frac{\cos 2x}{2^p} + \frac{\cos 3x}{3^p} + \ldots + \frac{\cos nx}{n^p} + \ldots$ converge uniformly on $\mathbb{R}$ for :

(a) $p \geq 1$

(b) $p > 1$

(c) $p \leq 1$

(d) $p < 1$

Q. 3. The series $1 + x + 2!x^2 + 3!x^3 + \ldots$:

(a) Has radius of convergence is 1

(b) Is nowhere convergent

(c) Is everywhere convergent

(d) None of the above

Q. 4. Interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n(n+1)}$ is :

(a) $-1 \leq x \leq 1$

(b) $-1 \leq x \leq 0$

(c) $-2 \leq x \leq 0$

(d) None of the above

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Q. 5. A linear operator A on $\mathbb{R}^n$ is invertible if and only if

(a) det [A] = 1

(b) det [A] = 0

(c) det [A] ≠ 0

(d) None of the above

Q. 6. Let m be a positive integer. If a vector space X is spanned by a set of m vectors, then

(a) dim X ≤ m

(b) dim X = m

(c) dim X ≥ m

(d) None of the above

Q. 7. If $A \in L(\mathbb{R}^n, \mathbb{R}^m)$ and if $x \in \mathbb{R}^n$, then

(a) A'(x) = A

(b) A'(x) = 0

(c) A'(x) = I

(d) None of the above

Q. 8. If $u = x + y + z$, $v = x - y + z$, $w = x^2 + y^2 + z^2 - 2xyz$ then $\frac{\partial(u,v,w)}{\partial(x,y,z)} =$

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(a) 4

(b) 4x

(c) 0

(d) None of the above

Q. 9. Let ω be k-form of class $\mathscr{C}''$ in some open set $E \subset \mathbb{R}^n$, then

(a) $d\omega = 1$

(b) $d\omega = 0$

(c) $d^2\omega = 1$

(d) $d^2\omega = 0$

Q. 10. The positively oriented boundary of Q^3 is :

(a) $[e_1, e_2, e_3] + [0, e_2, e_3] + [0, e_1, e_3] + [0, e_1, e_2]$

(b) $[e_1, e_2, e_3] - [0, e_2, e_3] + [0, e_1, e_3] + [0, e_1, e_2]$

(c) $[e_1, e_2, e_3] - [0, e_2, e_3] - [0, e_1, e_3] + [0, e_1, e_2]$

(d) $[e_1, e_2, e_3] - [0, e_2, e_3] + [0, e_1, e_3] - [0, e_1, e_2]$

(SECTION - B)

(Short Answer Type Questions)

Note : Attempt all questions. **4×5=20**

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Q. 1. Show that the series $\sum_{n=1}^{\infty} \frac{1}{1+n^x}$ converges uniformly in $[1, \infty)$

Q. 2. Show that

$$\frac{1}{2}(\tan^{-1}x)^2 = \frac{x^2}{2} - \frac{x^4}{4}\left(1+\frac{1}{3}\right) + \frac{x^6}{6}\left(1+\frac{1}{3}+\frac{1}{5}\right) - \cdots$$

$-1 < x \leq 1$

Q. 3. Let f maps a convex open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$. f be differentiable in E and there be a real number M such that $\|f'(x)\| \leq M$ for every $x \in E$. Then show that $\|f(b) - f(a)\| \leq M|b - a|$ for all $a \in E$, $b \in E$.

Q. 4. Describe the working rule of Lagrange's multiplier method for a function of several variables.

Q. 5. Suppose T is a 'ϕ'– mapping of an open set $E \subset \mathbb{R}^n$ into an open set $V \subset \mathbb{R}^m$, ϕ is a k-surface in E and ω is a k-form in V. Then prove that

$$\int_{T\phi} \omega = \int_{\phi} \omega_T$$

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(SECTION - C)

(Long Answer Type Questions)

Note : Attempt all questions. **8×5=40**

Q. 1. State and prove Cauchy criterion for uniform convergence.

OR

State and prove Weierstrass approximation theorem.

Q. 2. State and prove Riemann's theorem.

OR

State and prove Taylor's theorem on power series.

Q. 3. State and prove inverse function theorem.

OR

Let Ω be the set of all invertible linear operators on $\mathbb{R}^n$.

(a) If $A \in \Omega$, $B \in L(\mathbb{R}^n)$, and $\|B - A\| \|A^{-1}\| < 1$, then show that $B \in \Omega$.

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(b) Show that Ω is open subset is $L(\mathbb{R}^n)$ and the mapping $f : \Omega \to \Omega$ defined by $f(A) = A^{-1}$ for all $A \in \Omega$ is continuous.

Q. 4. Find the shortest distance from the point $\left(\frac{3}{2}, 0\right)$ to the parabola $y^2 = 4x$.

OR

Determine the maximum and minimum values of the function :

$$f(x, y) = x^2 + y^2 + z^2,$$

subject to the constraints $x^2 + y^2 + z^2 + xy + yz + zx = 1$ and $x + 2y - 3z = 0$.

Q. 5. State and prove Stokes' theorem.

OR

Suppose T is a $\mathscr{C}'$-mapping of an open set $E \subset \mathbb{R}^n$ into an open set $V \subset \mathbb{R}^m$, S is a

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$\mathscr{C}'$-mapping of V into an open set $W \subset \mathbb{R}^p$ and ω is a k-form in W, so that ω_S is a k-form in V and both $(\omega_S)_T$ and ω_{ST} are k-forms in E, where ST is defined by (ST)(x) = S(T(x)). Then prove that :

$$(\omega_S)_T = \omega_{ST}$$

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