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X–290

M. A./M. Sc. (First Semester)
EXAMINATION, Dec., 2013

MATHEMATICS

Paper First

(Advanced Abstract Algebra—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Prove that any *two* composition series of a finite group are solvable.

   (b) Show that if G is a commutative group having a composition series, then G is finite.

   (c) Let G be a group. Then show that, if G is solvable, then every subgroup and every homomorphic image of G are solvable. Conversely, if N is a normal subgroup of G such that N and G/N are solvable, then G is solvable.

Unit—II

2. (a) Let E be an algebraic extension of a field F containing an algebraic closure $\bar{F}$ of F. Then show that the ahead are equivalent :

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(i) Every irreducible polynomial in $F[x]$ that has a root in E splits into linear factors is E.

(ii) E is the splitting field of a family of polynomial in $F[x]$.

(iii) Every embedding σ of E in $\bar{F}$ that keeps each element of F fixed maps E onto E.

(b) If $f(x) \in F[x]$ be a nonconstant polynomial then prove that there exists an extension field E of F in which $f(x)$ has a root.

(c) If $f(x) \in F[x]$ is irreducible over F, then show that all roots of $f(x)$ have the same multiplicity.

Unit—III

3. (a) Prove that :

(i) Any finite field F with p^n elements is the splitting field of $x^{p^n} - x \in F_p[x]$.

(ii) Any *two* finite field with p^n elements are isomorphic.

(b) If K be a field extension of F, then show that that K is simple extension of F if and only if there are only a finite number of subfield of K containing F.

(c) Let F be a field. Show that there exists an algebraically closed field K containing F as a subfield.

Unit—IV

4. (a) Let E be a finite separable extension of a field F. Then show that the following are equivalent :

(i) E is normal extension of F

(ii) F is the fixed field of G(E/F)

(iii) $[E : F] = |G(E/F)|$

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(b) If K is splitting field of the polynomial $x^4 - 3x^2 + 4$ over Q, then find the Galois group of K over Q.

(c) Let E be a Galois extension of F and K be any subfield of E containing F. Then show that :

(i) $K = E_{G(E/K)}$

(ii) $H = G(E/E_H)$, for any subgroup H of G (E/F)

(iii) $[E : K] = |G(E/K)|, [K : F] =$ index of G (E/K) in G (E/F)

Unit—V

5. (a) Let F be a field of characteristic zero and E be splitting field of $x^n - a \in F[x]$. Then show that G(E/F) is a solvable group.

(b) Let $f(x)$ be a polynomial over a field F with no multiple roots. Show that $f(x)$ is irreducible over F if and only if the Galois group G of $f(x)$ is isomorphic to a transitive permutation group.

(c) Show that the cubic $f(x) = x^3 + 3ax^2 + 3bx + c$ over Q is solvable by radicals.

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