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Y–310

M. A./M. Sc. (First Semester) EXAMINATION, Dec., 2014

MATHEMATICS

Paper First

(Advanced Abstract Algebra—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) State and prove Jordan-Hölder theoerem.

   (b) Prove that, every subgroup and every homomorphic image of a nilpotent group if nilpotent.

   (c) Find all composition series of Z/(30). Verify that they are equivalent.

Unit—II

2. (a) Let $F \subseteq E \subseteq K$ be fields. If $[K : E] < \infty$ and $[E : F] < \infty$, then show that :

   (i) $[K : F] < \infty$;

   (ii) $[K : F] = [K : E].[E : F]$

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(b) Show that, a finite separable extension of a field F. is simple extension of F.

(c) Let $\omega = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right)$ and $u = \cos\left(\frac{2\pi}{n}\right)$, then show that :

$$\left[\mathbf{Q}\ (\omega) : \mathbf{Q}(u)\right] = 2$$

Unit—III

3. (a) Prove that, the multiplicative group of non-zero elements of a finite field is cyclic.

(b) Prove that :

$$\mathbf{Q}\left(\sqrt{2}, \sqrt{3}\right) = \mathbf{Q}\left(\sqrt{2} + \sqrt{3}\right)$$

(c) Show that, the prime field of a field F if either isomorphic to Q or to $Z/(p)$, for some prime p.

Unit—IV

4. (a) Let F and E be fields, then show that distinct embeddings of F into E are linearly independent over E.

(b) If K is splitting field of the polynomial $x^4 - x^2 + 1$ over Q, then find the Galois group of K over Q.

(c) Let E be a Galois extension of F and K be any subfield of E containing F. If K is normal extension of F, then show that :

$$G\ (K/F) = \frac{G\ (E/F)}{G\ (E/K)}$$

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Unit—V

5. (a) Let F be a field of characteristic zero and E be splitting field of $x^n - a \in F[x]$. Then show that G (E/F) is a solvable group.

(b) Let the field F has all nth root of unity (for some particular n) and suppose that $a \neq 0$ is in F. Let $x^n - a \in F[a]$ and let K be its splitting field over F, then show that :

(i) $K = F(u)$, where u is any root of $x^n - a$.

(ii) The Galois group of $x^n - a$ over F is abelian.

(c) Show that the cubic $f(x) = x^3 + 3ax^2 + 3bx + c$ over $\mathbf{Q}$ is solvable by radicals.

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800

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