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Y–311

M. A./M. Sc. (First Semester)
EXAMINATION, Dec., 2014

MATHEMATICS

Paper Second

(Real Analysis—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define uniform convergence of a series of functions. State and prove Cauchy's general principle of uniform convergence.

   (b) Test the uniformly convergence of the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$ in $0 < a \leq x \leq b < 2\pi$.

   (c) State and prove Weirstrass Approximation theorem.

Unit—II

2. (a) Define power series. State and prove Tauber's theorem.

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(b) Test the convergence of the following power series for $-1 \leq x \leq 1$:

$$\frac{x^2}{2} - \frac{x^3}{3}\left(1+\frac{1}{2}\right) + \frac{x^4}{4}\left(1+\frac{1}{2}+\frac{1}{3}\right) + \ldots\ldots$$

(c) Define absolute convergence of series. Prove that an absolute convergent series does not alter with any rearrangement of terms.

Unit—III

3. (a) Define linear transformation. Prove that a linear operator A on a finite dimensional vector space X is one to one iff A is onto.

(b) Give an example of implicit function. State and prove the Inverse Function theorem.

(c) Consider the mapping $f = (f_1, f_2)$ of R^5 into R^2 given by :

$$f_1(x_1, x_2, y_1, y_2, y_3) = 2e^{x_1} + x_2 y_1 - 4y_2 + 3$$

$$f_2(x_1, x_2, y_1, y_2, y_3) = x_2 \cos x_1 - 6x_1 + 2y_1 - y_3$$

Apply the implicit function theorem to compute $g'(3,2,7)$ taking $a = (0,1)$ and $b = (3,2,7)$.

Unit—IV

4. (a) Define stationary point. Determine the maximum and minimum values of the function $f(x,y) = x^2 + y^2 + \frac{3\sqrt{3}}{2}xy$ subject to the constraint $4x^2 + y^2 = 1$.

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(b) Find the maximum and minimum value of the function $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints $x^2 + y^2 + z^2 + xy + yz + zx = 1$ and $x + 2y - 3z = 0$.

(c) Explain Jacobians. Find Jacobian of the function $F = R^3 \rightarrow R^3$ with components :

$$y_1 = 5x_2$$

$$y_2 = 4x_1^2 - 2\sin(x_2 x_3)$$

$$y_3 = x_2 x_3$$

Unit—V

5. (a) Explain partitions of unity.

(b) Suppose T is a C'' mapping of an open set $E \subset R^n$ into an open set $V \subset R^m$, S is a C'' mapping of V into an open set $W \subset R^p$ and w is a k-form in W, so that W_S is a k-form in V and both $(W_S)_T$ and W_{ST} are k forms in E, where ST is defined, by $(ST)(x) = S(T(x))$. Then prove that :

$$(w_S)_T = w_{ST}$$

(c) State and prove Stoke's theorem.

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