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# Z–312

## M. A./M. Sc. (First Semester)
## EXAMINATION, Dec., 2015

### MATHEMATICS

### Paper Fourth

### (Complex Analysis—I)

*Time : Three Hours* ] [ *Maximum Marks : 80*

[ *Minimum Pass Marks : 16*

**Note :** Attempt any *two* parts from each question. All questions carry equal marks.

### Unit—I

1. (a) State and prove Cauchy's Integral formula.

   (b) Prove that :

$$\log(1-z) = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} + \ldots\ldots$$

$$+ (-1)^n \frac{(z-1)^n}{n} + \ldots\ldots$$

   (c) State and prove Liouville's theorem.

### Unit—II

2. (a) Every polynomial of degree $n$ has $n$ zeros.



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(b) Use Rouche's theorem to show that the equation $z^5+15z+1=0$ has one root in the disc $|z|<\frac{3}{2}$ and four roots in the annulus $\frac{3}{2}<|z|<2$.

(c) State and prove Schwarz lemma.

## Unit—III

3. (a) Show that :

$$\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}=\frac{2\pi}{\sqrt{a^2-b^2}},\ a>b>0$$

(b) Apply the calculus of residue, to prove that :

$$\int_0^{\infty} \frac{dx}{1+x^2}=\frac{\pi}{2}$$

(c) Use the method of contour integration to prove that :

$$\int_0^{\infty} \frac{\sin x}{x}dx=\frac{\pi}{2}$$

## Unit—IV

4. (a) To determine the bilinear transformation which transform three distinct points into three specified distinct points.

(b) To find all the bilinear transformation which maps the half plane $I(z)\geq 0$ onto the unit circular disc $|w|\leq 1$.

(c) Discuss the application of the transformation $w=z^2$ to the area in the first quadrant of the $z$-plane bounded by the axes and circles :

$$|z|=a,\ |z|=b,\ (a>b>0).$$



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## Unit—V

5. (a) State and prove Weierstrass's theorem.

   (b) State and prove Hurwitz's theorem.

   (c) State and prove open mapping theorem.

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