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Z–311

M. A./M. Sc. (First Semester)
EXAMINATION, Dec., 2015

MATHEMATICS

Paper Third

(Topology—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) (i) Write short notes on the following :

(A) Countable set

(B) Cardinal number

(C) Prove that $N \times N$ is countable.

(b) (i) If (X, T) is a topological space and $A, B \subset X$, then prove that :

(A) $A \subset B \Rightarrow \bar{A} \subset \bar{B}$

(B) $A \subset B \Rightarrow A^0 \subset B^0$, where A^0 denotes the interior of A.

(C) If (X, T) is a topological space and $A \subset X$, then prove that A is open if and only if it is a neighbourhood of each of its points.

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(c) Define derived set and dense set with an example. Show that a subset A of a topological spaces X is dense in X if and only if for every non-empty open subset B of X, $A \cap B \neq \phi$.

Unit—II

2. (a) Define the following with an example :

(i) Continuous function

(ii) Homeomorphism

(iii) Separable space

(iv) Lindeloff space

(b) (i) If a topological space (X, T) is second countable, then prove that every open cover of it has a countable subcover.

(ii) Show that compositions of continuous functions are continuous.

(c) Define first countable space and show that first countability is a hereditary property.

Unit—III

3. (a) Define the following with an example :

(i) T_1-space

(ii) Regular space

(iii) Normal sapce

(iv) Completely regular space

(b) (i) Prove that every subspace of a T_2-space is a T_2-space.

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(ii) If X is a regular space, then prove that for any $x \in X$ and any open set G containing x there exists an open set H containing x such that $\bar{H} \subset G$.

(c) (i) Show that every T_4-space is a T_3-space.

(ii) If X is a normal space, then prove that for any closed set C and any open set G containing C, there exists an open set H and a closed set K such that $C \subset H \subset K \subset G$.

Unit—IV

4. (a) (i) Show that every map from a compact space into a Hausdorff is closed.

(ii) Define the following with an example :

(A) Finite intersection property

(B) Compact space

(b) (i) Prove that a closed subspace of a compact space is compact.

(ii) Show that a continuous image of a sequentially compact set is sequentially compact.

(c) Define locally compact space and prove that every open subspace of a locally compact space is locally compact.

Unit—V

5. (a) Define countably compact space and prove that every countably compact metric space is second countable.

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(b) Define connected space with an example and prove that a subset of R is connected if and only if it is an interval.

(c) (i) Define the following with an example :

(A) Separated sets

(B) Component

(ii) Define locally connected space. If E be a component in a locally connected space X, then prove that E is open in X.

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