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A–313

M.A./M. Sc. (First Semester)
EXAMINATION, Dec., 2016

MATHEMATICS

Paper Fifth

(Advanced Discrete Mathematics–I)

Time : Three Hours] *[Maximum Marks : 80*

[Minimum Pass Marks : 16

Note : Attempt all questions.

Part—A

(Objective Type Questions)

Note : Choose the correct answer. 1 each

1. (S, *) is called semigroup if it satisfies the :
   (a) Closure, commutative
   (b) Associative, identity
   (c) Identity, inverse
   (d) Closure, associative
2. What is universal quantifiers ?
   (a) Symbol (∀) for all
   (b) Symbol (∃) for some
   (c) Symbol ∀ and ∃
   (d) None of the above

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3. Which of the following is not tautology ?

(a) $\neg(P \wedge Q) \Leftrightarrow (\neg P) \vee (\neg Q)$

(b) $P \wedge (Q \wedge S) \Leftrightarrow (P \wedge Q) \wedge S$

(c) $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

(d) $P \wedge (P \wedge Q)$

4. Which of the following laws are equivalent

(a) Distributive law

(b) De Morgan's law

(c) Associative law

(d) All of the above

5. $(M_1, *_1)$ and $(M_2, *_2)$ is called monoid homomorphism if :

(a) $f(a *_1 b) = f(a) *_2 f(b)$

(b) $f(e_1) = e_2$

(c) Both (a) and (b)

(d) None of the above

6. The direct product of any two semigroups is :

(a) Congruence relation

(b) Subgroup

(c) Semigroup

(d) Monoid homomorphism

7. $(T, *, e_m)$ is said to be submonoid if :

(a) $T \subseteq M$, where $(M, *, e_m)$ is monoid

(b) $T \subseteq S$, where S is semigroup

(c) Both (a) and (b)

(d) None of the above

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8. Every chain is distributive lattice means :

(a) $a \vee (b \wedge c) = a$

(b) $(a \vee b) \wedge (a \vee c) = a$

(c) $a \wedge (b \vee c) = a$

(d) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

9. A Boolean algebra is :

(a) Complemented distributive lattice

(b) Distributive lattice

(c) Uniquely distributive lattice

(d) Both (a) and (b)

10. $(L, \leq)$ is called lattice in which every pair of element $a, b \in L$ has :

(a) Greatest lower bound

(b) Least upper bound

(c) Both (a) and (b)

(d) None of the above

11. The join irreducible element of a lattice which succeed 0 is called :

(a) Join irreducible

(b) Product of sum

(c) Sum of product

(d) Atom

12. Every finite lattice is :

(a) Bounded

(b) Unbounded

(c) Undefined

(d) None of the above

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13. Conjunction normal form of $F(x, y, z) = xy + xz + xy'$ is :

(a) $x.\ y$

(b) $x.\ z$

(c) $x(1 + y)$

(d) $x(1 + y')$

14. Kleen's theorem states that :

(a) A set is regular iff it is not recognized by finite state automata

(b) A set is regular iff it is recognized by finite state automata

(c) Both (a) and (b)

(d) None of the above

15. A context free grammar contains productions of only the form $\alpha \rightarrow \beta$ where :

(a) $|\alpha| < |\beta|$

(b) $|\alpha| \leq |\beta|$

(c) $|\alpha| > |\beta|$

(d) $|\alpha| = |\beta|$

16. What is the length of string ?

(a) Equal to the number of string

(b) Equal to the number of symbol

(c) Equal to the number of null set

(d) Both (a) and (b)

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17. Complete disjunctive normal form in two variables x_1, x_2 is :

(a) 0

(b) 1

(c) Both (a) and (b)

(d) None of the above

18. Let $(S_1, *_1)$ and $(S_2, *_2)$ be two semi-groups, then the mapping $f : S_1 \to S_2$ is said to semi-homomorphism if :

(a) $f(a * b) = f(a) * f(b)$

(b) $f(a *_1 b) = f(a) *_1 f(b)$

(c) $f(a *_2 b) = f(a) *_2 f(b)$

(d) $f(a *_1 b) = f(a) *_2 f(b)$

19. Boolean identity $a \oplus (a' * b)$ is :

(a) $a * b$

(b) $a \oplus b$

(c) a

(d) $a * a'$

20. Absorption law is :

(a) $a \vee (a \vee b)$

(b) $a \wedge (a \wedge b)$

(c) $a \vee (a \wedge b)$

(d) None of the above

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Part—B

(Very Short Answer Type Questions)

Note : Very short answer in 2-3 sentences. 2 each

1. Define conditional.
2. Write rules of inference.
3. State basic homomorphism theorem.
4. Define congruence relation.
5. Write isotonicity property.
6. Write minterm Boolean form.
7. Write polish notation.
8. State pumping lemma.

Part—C

(Short Answer Type Questions)

Note : Short answer within $\leq$ 75 words. 3 each

1. Construct the truth table for the formula :

$$\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$$

2. Define semigroup and monoid.
3. Prove that $a \wedge a = a$ and $a \vee a = a$.
4. Draw the logic gate $(a.\bar{b}) + (\bar{a}.b)$.
5. Let $G = <\{0,1\}, \{S\}, S, \{S \rightarrow 11S, S \rightarrow 0\}>$ be a grammar, find L (G).
6. $f(a,b,c) = \Sigma(0,1,4,6)$, then simplify using Karnaugh map.
7. Write the following Boolean expression in an equivalent sum of product canonical form in variables x_1, x_2 and x_3 : $x_1 * x_2$.
8. Simplify the disjunction normal form $F(x,y,z) = xy' + xz + xy$.

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Part—D

(Long Answer Type Questions)

Note : Long answer using 150 words. 4 each

1. If $H_1, H_2 \ldots\ldots H_m$ and P imply Q, then $H_1, H_2 \ldots\ldots H_m$ imply $P \rightarrow Q$.

Or

Show that :

$$\left(\neg P \wedge \left(\neg Q \wedge R\right)\right) \vee \left(Q \wedge R\right) \vee \left(P \wedge R\right) \Leftrightarrow R .$$

2. Show that every finite lattice is bounded.

Or

For any $a, b \in L$, show that :

$$\left.\begin{array}{l} a * b = a * c \\ a \oplus b = a \oplus c \end{array}\right\} \Rightarrow b = c$$

3. Obtain the principal disjunctive normal form :

$$P \rightarrow \left[(P \rightarrow Q) \wedge \neg\left(\neg Q \vee \neg P\right)\right]$$

Or

Use the Karnaugh map representation to find a minimal sum of products expression of the following function :

$$f(a, b, c, d) = \Sigma(0, 1, 12, 13, 14, 15)$$

4. Let X be a set containing n elements and let X^* denote the free semigroup generated by X and let $(S, \oplus)$ be any other semigroup generated by any n generators, then show that there exists a homomorphism $I : X^* \rightarrow S$.

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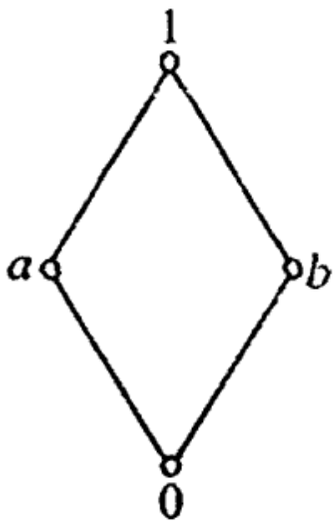
Or

Find the value of :

$$x_1 * x_2 * [(x_1 * x_4) \oplus x_2' \oplus (x_3 * x_1')]$$

for $x_1 = a, x_2 = 1, x_3 = b$ and $x_4 = 1$ where $a, b, 1 \in B$ and the Boolean algebra $(B, *, \oplus, ', 0, 1)$ is shown in the following using figure.



5. Show that the language

$$L(G_4) = \{a^n b a^n \mid n \geq 1\}$$

is generated by the grammar

$$G_4 = < \{S, C, \}, \{a, b\}, S, \phi >$$

where ϕ is the set of production :

$$S \rightarrow aCa, C \rightarrow aCa, C \rightarrow b$$

Or

Prove that the rank of any well formed Polish formula is 1 and the rank of any proper head of a Polish is greater than or equal to 1.

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