Roll No. .....

ID–313

M. Sc. (First Semester)

EXAMINATION, Dec.–Jan., 2024-25

MATHEMATICS

Paper Fifth

(Advanced Discrete Mathematics–I)

Time : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 16

Note : Answer any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define Tautology and show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

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(b) Show that

$$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$$

is a tautology.

(c) Define the following :

(i) Semigroup

(ii) Monoid

(iii) Propositional logic

(iv) Quantifier

Unit—II

2. (a) Define semigroup Homomorphism and let (S, *), (T, Δ) and (V, ⊕) be semigroup and $g : S \rightarrow T$ and $h : T \rightarrow V$ be semigroup homomorphism, then prove that $h \circ g : S \rightarrow V$ is a semigroup homomorphism from (S, *) to (V, ⊕).

(b) Let X be a set containing n and X^* denoted the free semigroup generated by X and let (S, ⊕) be any other semigroup generated by n generator, then there exists a homomorphism $I : X^* \rightarrow S$.

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(c) Define subsemigroup and prove that (M, *) be a monoid then there exists a subset $T \subseteq M^{(M)}$ such that (M, *) is isomorphism to the monoid (T, 0).

Unit—III

3. (a) Define distributive lattice; for any $a, b, c \in L$, show that

$(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c$ where (L, *, $\oplus$) is a distributive lattice.

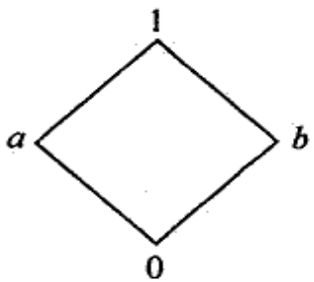
(b) Find the value of :

$x_1 * x_2 * [(x_1 * x_4) \oplus x_2^1 \oplus (x_3 * x_2^1)]$

for $x_1 = a_1, x_2 = 1, x_3 = b$ and $x_4 = 1$, where

$a, b, 1 \in B$ and the Boolean algebra

(B, *, $\oplus$, ′, 0, 1) is shown in figure.



(c) Let L = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24} be ordered by the relation | where $x \mid y$ means 'x divides y'. Show that D_{24} the set of all divisors of the integer 24 of L is a sublattice of the Lattice (L, 1).

Unit—IV

4. (a) Write the following Boolean expressions in an equivalent sum-of-product form in three variables x_1, x_2 and x_3 :

(i) $x_1 * x_2$

(ii) $(x_1 \oplus x_2)' * x_3$

(iii) $x_1 \oplus x_2$

(b) Use the Karnaugh map representation to find a minimal sum of product expression of the following function :

$f(a, b, c, d) = \Sigma (0, 1, 4, 5, 9, 11)$

(c) Define the following :

1. (i) Direct product

   (ii) Sum of product canonical forms

2. Switching theory

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Unit—V

5. (a) Define grammar and let G = ({0, 1}, {S}, S, {S → 11S, S → 0}) be a grammar. Determine L(G).

(b) Define the following :

(i) Regular set

(ii) Polish Notations

(iii) Notions of Syntax Analysis

(iv) The Reverse Polish Notation

(c) Define language and construct a grammar for the language $L = \{a^i b^{2i} : i > 1\}$.

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