PF-157

M.A./M.Sc. Mathematics

1st Semester Examination, Dec., 2022

Paper - V

Advanced Discrete Mathematics-I

Time : Three Hours] [*Maximum Marks* : 80

[*Minimum Pass Marks* : 29

Note : Answer from **both** the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions : 1×10

(*a*) The truth value of statement $[p \wedge (p \Rightarrow q)] \Rightarrow q$ when p is true and q is false , is ..... .

(*b*) The dual of statement $(p \wedge q) \vee T$ is ..... .

(*c*) The rule of detachment is also known as ..... .

(*d*) The quantifier is the statement 'some people are crazy' is ..... .

(*e*) The greatest element of poset $(P(x), \subseteq)$ is ....., where $P(x)$ is power set of $X = \{a, b, c\}$.

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(f) The complement of 3 in Lattice (L, l) is ..... where $L = \{1, 2, 3, 4, 6, 12\}$ with relation 'l'. (divides)

(g) $(N, +)$ is infinite cyclic monoid generated by ..... .

(h) The identity element of monoid $(N, *)$ is ..... where N is set of natural number and '$*$' is $x * y = \max\{x, y\} \forall x, y \in N$.

(i) Boolean algebra can not have ..... elements.

(j) The Karnaugh map is a ..... representation of truth table of the Boolean algebra.

2. Answer the following short answer type questions :

(a) Show that set $\{\wedge, \vee\}$ is not functionally complete.

(b) Show that Poset $\{L, l\}$ where $L = \{1, 2, 3, 4, 5\}$ is not lattice.

(c) Prove that in Boolean algebra B. $1' = 0$

(d) Write all minimal Boolean functions of three variables x, y, z.

(e) Prove that the identity element in any monoid is unique.

Section-B

Answer the following questions :

3. (a) Prove that

$$\sim(p \wedge q) \rightarrow [\sim p \vee (\sim p \vee q)] \equiv p \vee q$$

without constructing truth table.

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(b) Prove that

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

OR

(a) Show that the following argument is valid

$$\begin{array}{l} p \\ p \rightarrow q \\ \underline{q \rightarrow r} \\ r \end{array}$$

(b) Negate the following statements :

(i) $\forall x (|x| = x)$

(ii) $\exists x \left(x^2 < 0\right)$

(iii) $\exists x\, P(x) \vee \forall y\, Q(y)$

4. Prove that a modular lattice is distributive if and only if it has a sublattice isomorphic to the lattice M_5. https://www.abvvonline.com

OR

If $(L, \leq)$ is a lattice, then for any $a, b, c \in L$ prove that .

(a) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

(b) $a \wedge (a \vee b) = a$

5. In Boolean algebra $(B, +, \cdot)$ Prove that

(a) $a(bc) = (ab)c \; \forall a, b, c \in B$

(b) $a + (b + c) = (a + b) + c \; \forall a, b, c \in B$

OR

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Use the Karnaugh map to find a minimal form of the following :

(a) $f \equiv xyz'w' + xyz'w + xy'zw + xy'zw' + x'y'zw + x'y'zw' + x'yw'z'$

(b) $f \equiv xyz' + xy'z' + x'y'z' + x'y'z + x'yz + x'yz'$

6. (a) State and prove fundamental theorem of semigroup homomorphism.

(b) Prove that the set of all semigroup endomorphism of a semigroup is a semigroup under the operation of composition of functions.

OR

(a) Prove that every finite semigroup has an idempotent element.

(b) Let $S = \{1, 2, 3, 6, 12\}$ and operation '*' be defined by $a * b$ = greatest common divisor of a and b. Show that $(S, *)$ is a semigroup.

7. Let $B = \{1, 2, 5, 7, 10, 14, 35, 70\}$. Then prove that $(B, +, \cdot)$ is Boolean algebra where '+' and '.' is defined by

$$a + b = \text{LCM}(a,b) \forall a,b \in B$$

$$a \cdot b = \gcd(a,b) \forall a,b \in B$$

OR

Prove that for any two elements a, b of a Boolean algebra $(B, +, \cdot)$

(a) $a + b = \text{lub}\ \{a, b\}$

(b) $a \cdot b = \text{glb}\ \{a, b\}$

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