PD-156-S.E.-CV-19

M.A./M.Sc. MATHEMATICS (1st Semester)

Examination, Dec.-2020

Paper-IV

COMPLEX ANALYSIS-I

Time : Three Hours] **[Maximum Marks : 80**

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. **Answer the following questions:-** 1x10=10

   (a) **Who was the first which obtained the definite integral by replacing a real variable by a complex variable.**

   (b) **If** $f(z)$ **is analytic within and on the boundary** c **of a simply connected region and** $'a'$ **is any point within then** $\int_c \frac{f(z)}{(z-a)^3} dz = \cdots \ldots \ldots$

   (c) **Write the value of** $\int_c \frac{z^2}{(z-a)^3} dz$ **where** c **is circle** $|z| = 3$

   (d) **If** $f(z)$ **is analytic within and on a circle** c **defined by** $|z - z_0| = 2$ **and** $|f(z)| \leq K$ **one** C **then** $|f^{(iv)}(z_0)| \leq \cdots \ldots \ldots \ldots \ldots$

   (e) **Zero of an analytic functions are.....**

   (f) **A function which has no singularity.....is constant**

   (g) **How many roots of** $z^8 - 4z^4 + z^2 - 1 = 0$ **lie inside the circle** $|z| = 1$

   (h) **If** $z = a$ **is only singularity of an analytic function** $f(z)$ **inside a closed curve** C **then the residue of** $f(z)$ **at** $z = a$ **is.....**

   (i) **Write the fixed point of** $\frac{z-1}{z+2}$

   (j) **Write the condition that transformation** $w = \frac{az+b}{cz+d}$ **is normal.**

2. **Answer the following questions:-** 2x5=10

   (a) **Evaluate** $\int_c \frac{2z+1}{z^2+2}$ **where** c **is** $|z| = \frac{1}{2}$

   (b) **Expand** $\frac{1}{(z-1)(z-2)}$ **for the rigion** $|z| > 2$

   (c) **Find the residue of** $\frac{z^2+z+1}{z(2z+1)(z+2)}$ **at** $z = -\frac{1}{2}$

   (d) **Find the normal form of** $w = \frac{z}{2-z}$

   (e) **Distinguish between pole and essential singularity.**

Section-B 12x5=60

Answer all questions.

3. **State and prove Laurent's theorem.**

   OR

   State and prove Cauchy Goursat's theorem.

4. **Find the expansion of** $\frac{1}{(z^2+1)(z^2+2)}$ **in power of** z **when**

   (i) $|z| < 1$ (ii) $1 < |z| < \sqrt{2}$ (iii) $|z| > \sqrt{2}$

   OR

   State and prove maximum modus principle.

5. **Show that one root of the equation** $z^4 + z + 1 = 0$ **Lies in the first quadrant.**

   OR

   If $f(z)$ **is analytic within and on a closed contour** C **except at a finite number of poles and has no zero on** C **then** $\frac{1}{2\pi i}\int_c \frac{f'(z)}{f(z)} dz = N - P$

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Where N is the number of zero and P is the number of poles inside C (a pole or zero of order m being counted m times)

6. Find the value of following integrals by using calculus of residues

(a) $\int_0^{\pi} \frac{1+2\cos\theta}{5+3\cos\theta} d\theta$

(b) $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$

OR

Apply the method of calculus of residues to prove that

(a) $\int_0^{\infty} \frac{dx}{(1+x^2)^2} = \frac{3\pi}{8}$

(b) $\int_0^{\infty} \frac{\cos x}{x^2+a^2} dx = \frac{\pi e^{-a}}{2a}$ $\quad (a > 0)$

7. Find the bilinear transformation which transforms half plane $R(z) \geq 0$ onto the unit circular disc $|w| \leq 1$

OR

State and prove sufficient condition for $w = f(z)$ to represent a conformal mopping.

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