PF-154

M.A./M.Sc. Mathematics

1st Semester Examination, Dec., 2022

Paper - II

Real Analysis-I

Time : Three Hours] [*Maximum Marks* : 80

[*Minimum Pass Marks* : 29

Note : Answer from **both** the Sections as directed. **The figures in the right-hand margin indicate marks.**

Section-A

1. Answer the following questions : 1×10

(a) Write any one property of *RS*-integral.

(b) If P^* is refinement of a partition P, then what is relation between $U(P^*, f, \alpha)$ and $U(P, f, \alpha)$?

(c) Write Schwarz's inequality.

(d) What is rectifiable curve in R^K ?

(e) Write the definition of uniform convergence of a series of functions.

(f) We know that "$f_n \to f$ uniformly on E if and only if $M_n \to 0$ as $n \to \infty$." What does M_n stands for ?

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(g) What is radius of convergence of a power series ?

(h) What does invertible linear operator mean ?

(i) What is linear transformation ?

(j) What does $L(X)$ stand for ?

2. Answer the following questions : 2×5

(a) Explain integration of vector valued function.

(b) Write a Cauchy criteria for uniform convergence of a sequence of functions.

(c) Prove that series of functions $\sum \frac{\sin nx}{n^2}$ converges uniformly.

(d) Find the radius of convergence of power series $\sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n+1}}$.

(e) Write a statement of implicit function theorem.

Section-B

Answer the following questions : 12×5

3. (a) Prove that $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\epsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$.

(b) If f is monotonic on $[a, b]$ and if α continuous on $[a, b]$, then prove that $f \in R(\alpha)$.

OR

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(a) State and prove fundamental theorem of calculus.

(b) If f maps $[a, b]$ into R^K and if $f \in R(\alpha)$ for some monotonically increasing function α on $[a, b]$, then prove that $|f| \in R(\alpha)$ and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$.

4. (a) State and prove Cauchy criteria for uniform convergence for sequence of functions.

(b) For what value of 'a' the series $\sum_{n=1}^{\infty} \frac{n^a x}{1+n^2x^2}$ is convergent in $0 < x < 1$.

OR

(a) Suppose $\{f_n\}$ is a sequence of functions defined on E and suppose $|f_n(x)| \leq M_n (x \in E, n = 1, 2, 3, ...)$. Prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

(b) State and prove Abel's test for uniform convergence. https://www.abvvonline.com

5. Let $\{f_n\}$ be sequence of differentiable functions on $[a, b]$ such that it converges at least at one point $x_0 \in [ab]$. If the sequence of differentials $\{f_n'\}$ converges uniformly to G

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on $[a, b]$, then prove that the given sequence $\{f_n\}$ converges uniformly on $[ab]$ to $f(x)$ and $f'(x) = G(x)$.

OR

State and prove Stone-Weierstrass theorem.

6. If a power series $\sum a_n x^n$ converges for $|x| < R$ and let us define a function $f(x)$, $f(x) = \sum a_n x^n$, $|x| < R$, then prove that $\sum a_n x^n$ converges uniformly on $[-R + \epsilon, R - \epsilon]$, no matter which $\epsilon > 0$ is chosen and that the function f is continuous and differentiable on $(-R, R)$ and $f'(x) = \sum n a_n x^{n-1}$, $|x| < R$.

OR

State and prove Abel's theorem for power series.

7. (*a*) Prove that a linear operator A on a finite dimensional vector space X is one to one if and only if the range of A is all of X.

(*b*) Suppose f maps a convex open set $E \subset R^n$ into R^m, f in differentiable in E and there is a real number M such that $\|f'(x)\| \leq M$ for every $x \in E$. Then prove that $|f(b) - f(a)| \leq M |b - a|$ for all $a \in E$, $b \in E$.

OR

State and prove Inverse Function theorem.

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