PF-155

M.A./M.Sc. Mathematics

1st Semester Examination, Dec., 2022

Paper - III

Topology - I

Time : Three Hours] [*Maximum Marks* : 80

[*Minimum Pass Marks* : 29

Note : Answer from **both** the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Define the following : 1×10

(a) Co-finite topology on a set X.

(b) Usual topology in R.

(c) Base for a topology.

(d) Neighbourhood of a point with example.

(e) Sub-space of a topological space.

(f) Limit point of a set with example.

(g) Second countable space.

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(h) Separable space.

(i) Continuity in a topological space.

(j) Closure of a set.

2. Answer the following questions : 2×5

(a) Show that every open interval on R is a μ-open set

(b) Prove that

$$\overline{A \cap B} \subset \overline{A} \cap \overline{B}$$

(c) Give an example to show that every T_1-space is not a T_2-space.

(d) Show that every discrete space is first countable.

(e) Prove that constant functions are always continuous.

Section-B

Answer all questions : 12×5

3. Prove that the intersection of two topologies on X is also a topology on X but union of two topologies is not necessarily a topology.

OR

(a) Show that a subset of a topological space is open iff it is a neighbourhood of each of its points.

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(b) Let X be a topological space. If F is a closed subset of X and $x \in F^1$, then prove that there exists a neighbourhood N of x such that $N \cap F = \phi$

4. (a) Let A, B be subsets of a topological space, then show that

(i) $D(\phi) = \phi$

(ii) $A \subset B \Rightarrow D(A) \subset D(B)$

(iii) $D(A \cap B) \subset D(A) \cap D(B)$

(iv) $D(A \cup B) = D(A) \cup D(B)$

(b) Prove that every finite subset of R is μ-closed.

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OR

Let X be any set and let c be the Kuratowski closure operator on X. Then show that there exists a unique topology $\Im$ on X such that for each $A \subset X$, $c(A)$ coincides with $\Im$-closure of A i.e. $c(A) = \overline{A}$.

5. Show that a mapping f from a space X into another space Y is continuous iff $f(\overline{A}) \subset \overline{f(A)}$ for every $A \subset X$.

OR

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(g) Prove that $\overline{A} = A \cup D(A)$

(b) Prove that composite of two continuous functions in a continuous function.

6. (a) Let $f: R \to R$ defined by $f(x) = x^2$, $\forall x \in R$, find whether the mapping is μ-μ continuous.

(b) Let X and Y be topological spaces, then show that a mapping $f: X \to Y$ is continuous iff the inverse image under f of every open set in Y is open in X.

OR

State and prove Tietz extension theorem.

7. (a) Show that a closed subspace of a normal space is normal.

(b) Prove that the property of being a T_2-space is a topological property

OR

(a) Show that the space (R, μ) is T_3.

(b) Prove that every second countable space is separable.

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