PE-153

(511) M.A./M.Sc. Mathematics (First Semester)
EXAMINATION, DEC.-2021
Paper-I
Advanced Abstract Algebra (I)

Time : Three hours] **[Maximum Marks : 80**

नोट :-दोनों खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित हैं।

Note :-Answer from both the sections as directed. The figures in the right-hand margin indicate marks.

[Section-A]

1 Answer the following questions : **1×10=10**

(a) If f and g are two cyclic permutations then fg = ...

(b) The number of even permutations in 5 symbols is..... .

(c) If $f : G \to G'$ is an isomorphism, $0(G) = n$, $a \in G$ and $o(a) = m$ then $o[f(a)]$ = ...

(d) If G is an infinite cyclic group then o[Aut(G)] = ...

(e) If $a \in Z(G)$ then C(a) = ...

(f) Is the symmetric group S_6 a solvable group ?

(g) Every solvable group is nilpotent. [True/False]

(h) Let (I, +, ·) be the commutative ring of integers then ideal (5) is a prime ideal. [True/False]

(i) Ring of integers is not a principal ideal ring. [True/False]

(j) The ideal (7) of ring of integers is.....ideal of I. [Maximal/Minimal]

2 Answer the following questions : **2×5=10**

(a) Show that every subgroup of an abelian group is normal subgroup.

(b) Define Nilpotent group.

(c) If H is a subgroup of G and N is a normal subgroup of G then show that $H \cap N$ is a normal subgroup of H.

(d) Show that every abelian group is solvable.

(e) Show that commutator subgroup G' of group G is a normal subgroup of G.

[Section-B]

Answer all questions. **12×5=60**

3 Show that the set I(G) of all inner automorphism is a normal subgroup of Aut(G) and is isomorphic to G/Z, where Z is the centre of G.

OR

State and prove second law of isomorphism.

4 Show that every subgroup of a solvable group is solvable.

OR

(P.T.O.)

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Show that every homomorphic image of nilpotent group is nilpotent.

5 Let A and B be R-submodules of R-modules M and N respectively. Then show that :

$$\frac{M \times N}{A \times B} \simeq \frac{M}{A} \times \frac{N}{B}.$$

OR

Let R be a ring with unity. Show that an R-module M is cyclic iff $M \cong \frac{R}{I}$ for some ideal I of R.

6 State and prove Schur's lemma.

OR

Show that every finite P-group is nilpotent.

7 Show that an ideal S in the ring of integers I is maximal iff it is generated by prime integers.

OR

Let H be a proper normal subgroup of G. Then show that H is maximal normal subgroup of G iff quotient group G/H is simple.

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