PE-155

(511) M.A./M.Sc. MATHEMATICS (First Semester)

EXAMINATION, DEC.-2021

TOPOLOGY (I) (Paper - III)

Time : Three hours] **[Maximum Marks : 80**

नोट– दोनों खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित हैं।

Note : *Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.*

Section–A

1. Answer the following questions : **1×10=10**

(1) Define topology in terms of open set.

(2) Define neighbourhood of a point in a topological space.

(3) Give the definition of open sets and closed sets.

(4) Which of the following subsets of R are U-neighbourhood of 1 ?

(a) $]0, 2[$ (b) $[1, 2]$ (c) $[1, 2[$

Explain your answer.

(5) Define local base at a point x of X. Where (X, T) in the topological space.

(6) Define first countable space with example.

(7) If (X, T) is topological space and $A \subset X$, then define limit point of A.

(8) Define continuity in topological space.

(9) Define T_0 space.

(10) Give the definition of Homeomorphism.

2. Answer the following questions : **2×5=10**

(1) Show that intersection of two topologies is a topology.

(2) Show that every T_2-space is a T_1-space.

(3) Show that every discrete space is a T_0-space.

(4) Define Normal spaces.

(5) Give the statement of Titze-Extension Theorem.

Section–B

Answer all the questions : **12×5=60**

3. (1) Let $\{T_\lambda : \lambda \in \wedge\}$ where $\wedge$ is an arbitrary set be a collection of topologies for X. Then show that the intersection $\cap \{T_\lambda : \lambda \in \wedge\}$ is also a topology for X.

(2) If F_1 and F_2 be two closed subsets of a topological space X. Then show that $F_1 \cup F_2$ is a closed set.

OR

Let X be a non-empty set and F a family of subsets of X such that :

$(F_1): \phi \in F, X \in F$

$(F_2): F_1, F_2 \in F \Rightarrow F_1 \cup F_2 \in F$

$(F_3): F_\lambda \in F \; \forall \lambda \in \wedge \Rightarrow \cap \{F_\lambda : \lambda \in \wedge\} \in F$

[P.T.O.]

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then show that there exists a unique topology for X such that the T-closed subsets of X are precisely the members of F.

4. (1) Show that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.

(2) Let X be a topological space, and for each $x \in X$ let $N(x)$ be the collection of all nods of x, then show that :

(a) $N \in N(x) \Rightarrow x \in N$

(b) $N \in N(x); M \supset N \Rightarrow M \in N(x)$

(c) $N \in N(x), M \in N(x) \Rightarrow N \cap M \in N(x)$

OR

(1) Let (X, T) be a topological space, then show that a sub collection B of T is a base for T iff every T-open set can be expressed as the union of members of B.

(2) Give the definition of Base for topology.

5. (1) Let X be a topological space, and let A be a subset of X. Then show that A is closed if and only if $D(A) \subset A$.

(2) Prove that : $$\overline{A} = A \cup D(A)$$

OR

Let (X, T) be a topological space and A, B be any subsets of X. Then show that :

(a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

(b) $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$

(c) $\overline{\overline{A}} = \overline{A}$

6. Let X and Y be topological spaces. A mapping, show that $f : X \to Y$ is continuous if and only if the inverse under f of every open set in Y is open in X.

OR

Show that : A mapping f of a space X onto another space Y is continuous if and only if : $$\overline{f^{-1}(A)} \subset f^{-1}(\overline{A}) \text{ for every } A \subset Y$$

7. (1) Show that Normality is a topological property.

(2) Show that every T_3-space is a T_2-space.

OR

State and prove Urysohn's lemma.

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